The Cybernetix Case Study. Probabilities and Non-determinism.

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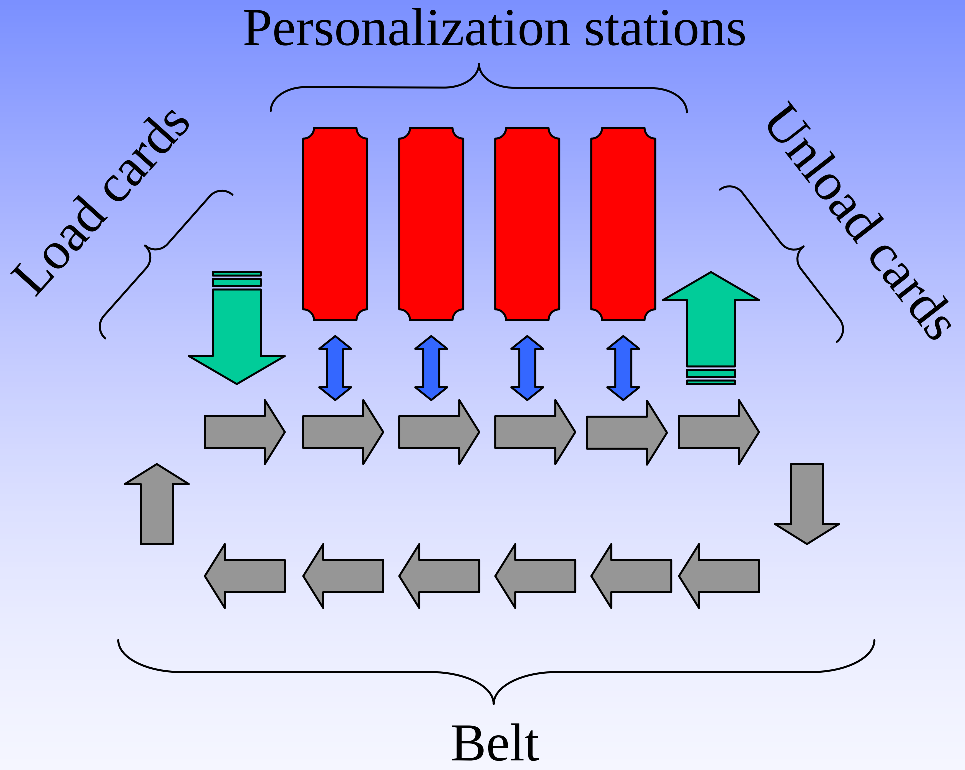
Outline

- The Cybernatix Case Study
- The main interest
- Prism tool
- Models
- Conclusions

The Cybernatix Case Study



Main interests

- Involve *probabilistic* and/or *non-deterministic* failures
- Types of failures
  - A card can be broken
  - A personalization station can be broken

P(M of N cards are broken) = ?

Prism tool

- A probabilistic model checker
- Supports the following probabilistic models:
  - **DTMC** – discrete time Markov chain
  - **MDP** – Markov decision process
  - CTMC – continuous-time Markov chain
- Supports the following temporal logics
  - **PCTL** (probabilistic computation tree logic) – for DTMCs and MDPs
  - CSL (continuous stochastic logic) – for CTMCs

PCTL and properties under consideration

The syntax of PCTL:

$$\phi ::= true \mid a \mid \phi \wedge \phi \mid \neg \phi \mid \mathrm{P}_{\triangleright\triangleleft p}[\varphi]$$

$$\varphi ::= \mathrm{X}\phi \mid \phi \bigcup^{\leq k} \phi \mid \phi \bigcup \phi$$

The verified property:

$$\mathrm{P}_{=?}\left[\, true \bigcup M \;\; of \;\; N \;\; cards \; are \; broken \right]$$ - for DTMCs

$$\left.\begin{array}{l} \mathrm{P}_{\min=?}\left[\, true \bigcup M \;\; of \;\; N \;\; cards \; are \; broken \right] \\ \mathrm{P}_{\max=?}\left[\, true \bigcup M \;\; of \;\; N \;\; cards \; are \; broken \right] \end{array}\right\}$$ - for MDPs

The Prism language

Simple state-based language, based on the Reactive Models formalism of Alur and Henzinger.

Fundamental concepts:

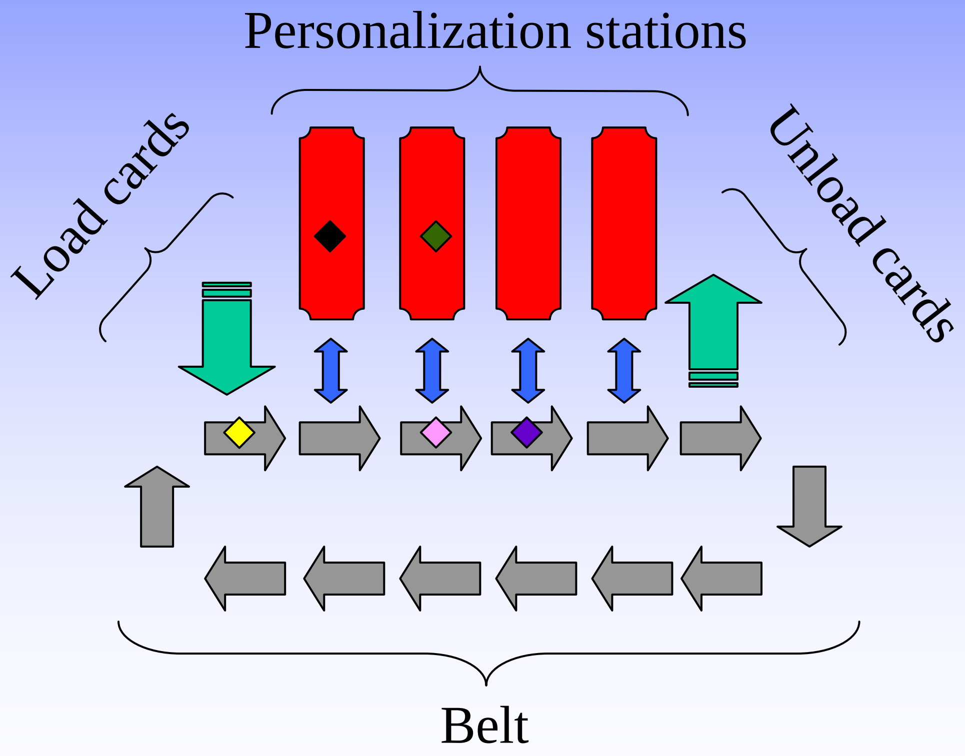
- Modules
- Variables
- Parallel compositions of modules

The behaviour of each module is described by commands:

$$[\ label\ ]\ g \rightarrow \lambda_1 : u_1 + \ldots\ \lambda_n : u_n ;$$

Super Single Mode

- Do everything as fast as you can, and leave free space for personalized cards.
- Give uniform loading of stations



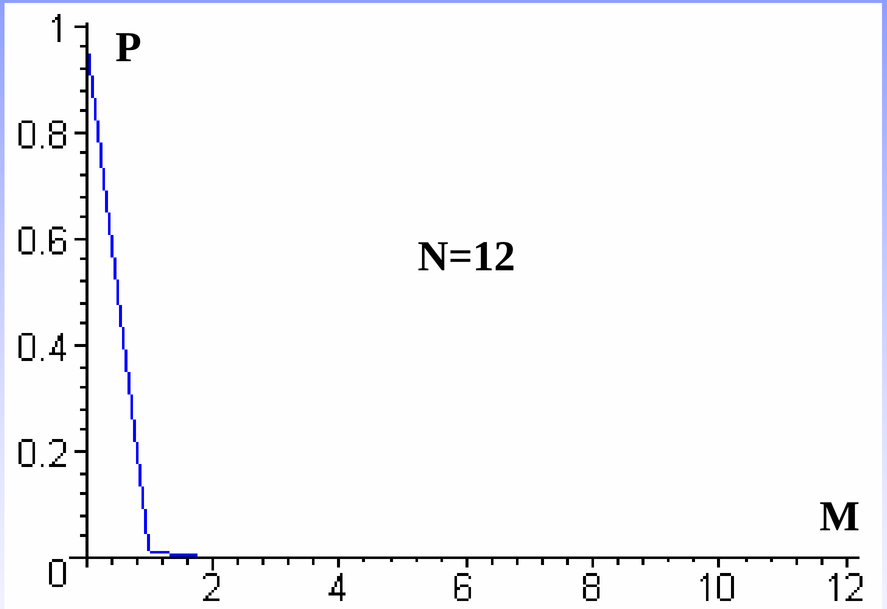
Simple failure Model

The model: Each card can be broken while personalization with the probability (1-p). In our case p = 0.999.

Analytical solution:

- All card breakings are independent
- The probability of breaking a card is (1-p)

$$p^{N-M}(1-p)^{M} \quad , \quad C_N^M = \frac{N!}{M!(N-M)!}$$



$$P(M\ of\ \ N\ cards\ are\ broken) = C_N^M\, p^{N-M}(1-p)^{M}$$

Type I discrete Weibull distribution

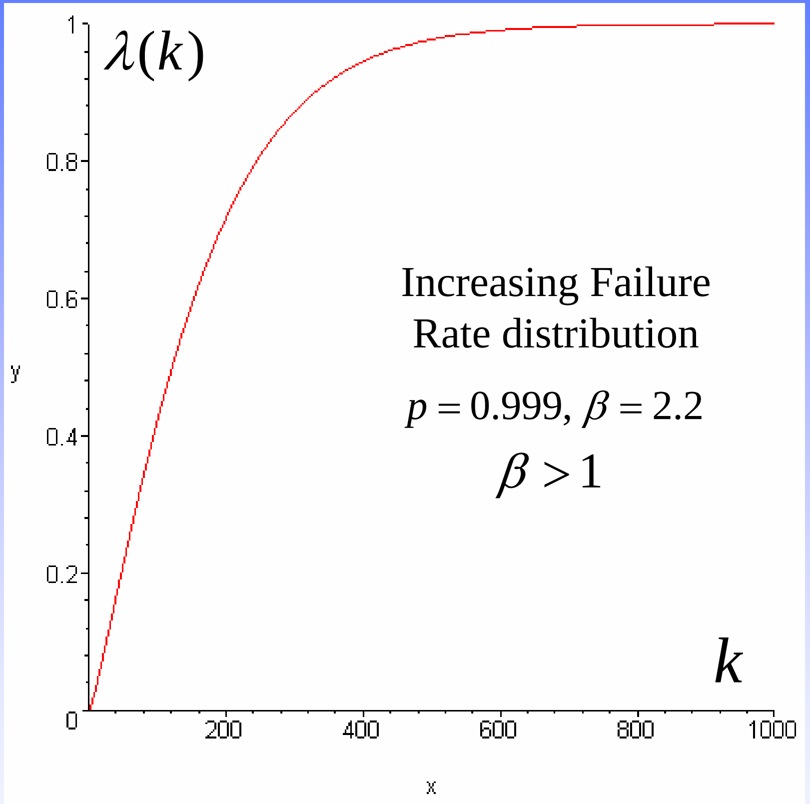
$R(k) = p^{k^{\beta}}$ - reliability function

$\lambda(k) = 1 - p^{k^{\beta}-(k-1)^{\beta}}$ - failure rate

$p \in \left]0,1\right[,\ \beta > 0,\ k \in N^{*}$

$P(k) = P(K = k)$ - probability of failure at demand *k*

$R(k) = P(K > k)$ - probability not to fail during *k* demands



Increasing Failure Rate distribution

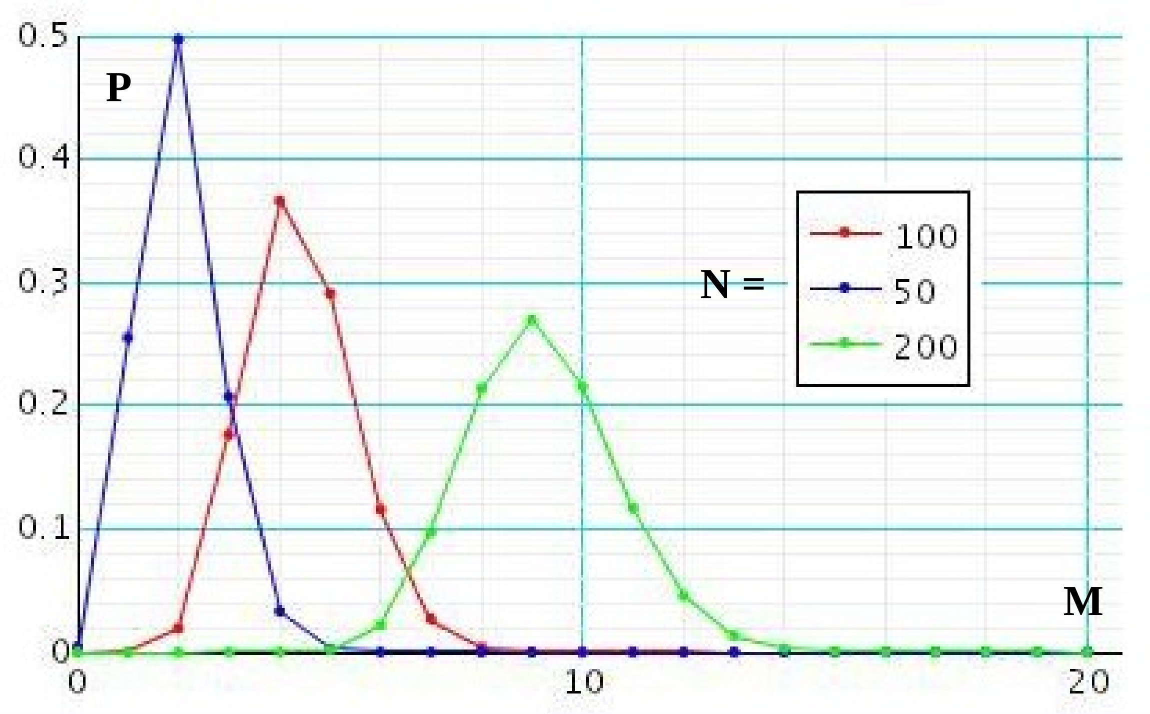
$p = 0.999,\ \beta = 2.2$

$\beta > 1$

$\lambda(k) = P(K = k \mid K \geq k) = \dfrac{P(k)}{R(k-1)}$ - probability to fail at demand k if it did not fail before.

Simple Failure Model with Weibull distribution of failures

The model: Each card can be broken while personalization with the probability $\lambda(k)$ with $p = 0.999, \beta = 2.2$ where k is the number of cards, correctly personalized, by the given station since it broke card for the last time.



One station results generalization

- Uniform stations loading (SSM)
- Independent station failures

- ~~Time~~
- ~~Loading station~~
- ~~Unloading station~~
- ~~Belt~~
- ~~4 personalization stations~~

$$P_T^R\left(M \;\; of \;\; N \;\; cards \;\; are \;\; broken\right) = \sum_{M_1+\ldots+M_N=M} \prod_{i=1}^{R} P_T^1\left(M_i \;\; of \;\; \frac{N}{R} \;\; cards \;\; are \;\; broken\right)$$

N - the total amount of cards

R - the number of stations, is the divisor of N

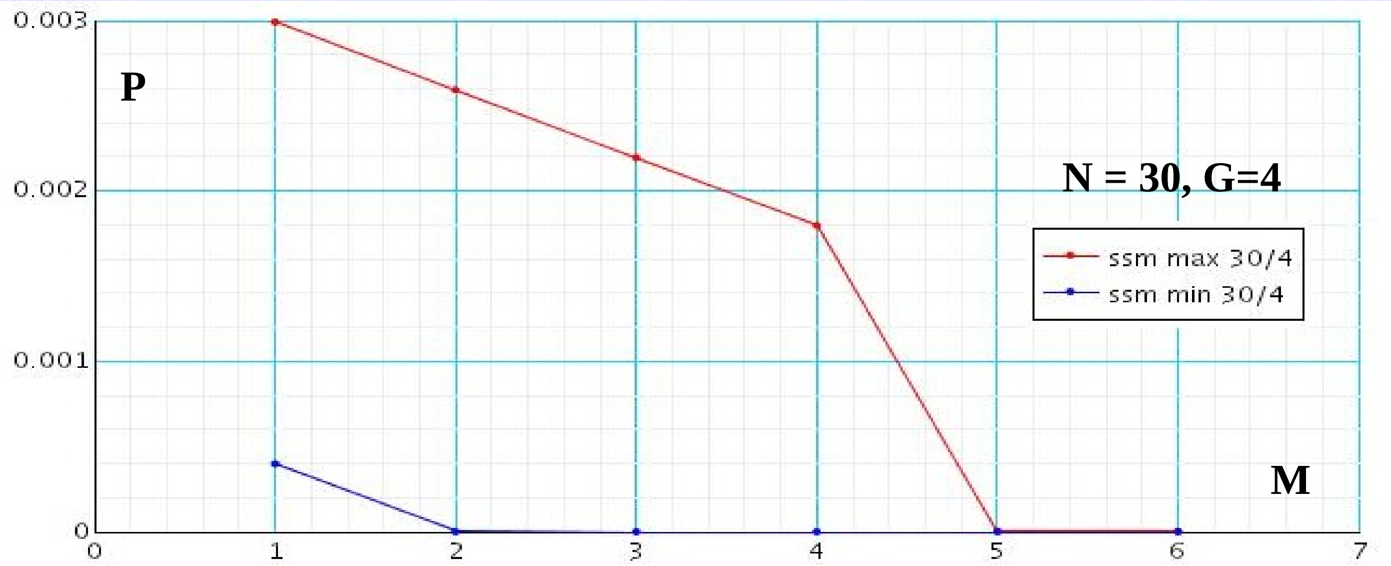
$T \in \{\min, \max, \_\}$

P_T^1 - the probability for one station

Model with the non-deterministic number of broken cards

The model:

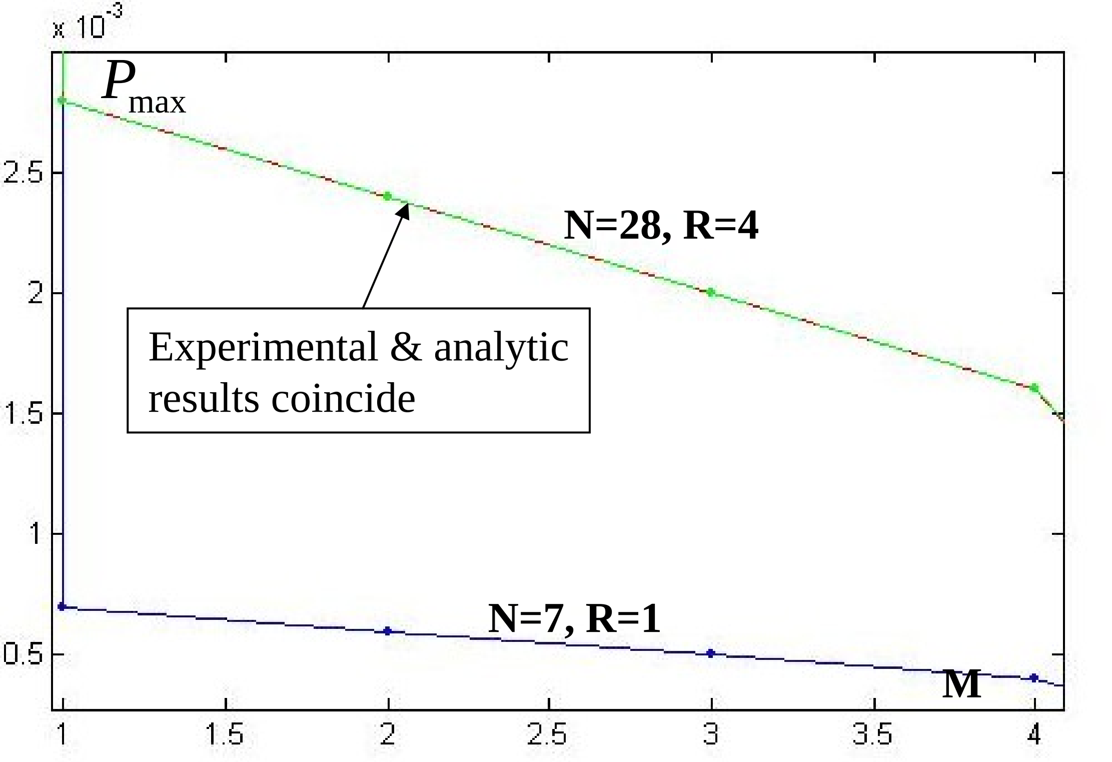
- 4 personalization stations
- Station can break each time it takes a new card with probability (1-p), p=0.999
- After breaking it spoils any number of cards within [1,G], G=4



Non deterministic model for one card and it's extension

The model:

- One station, No time
- Station can break each time it takes a new card with probability (1-p), p=0.999
- After breaking it spoils any number of cards within [1,G]
- No belt, loading and unloading stations

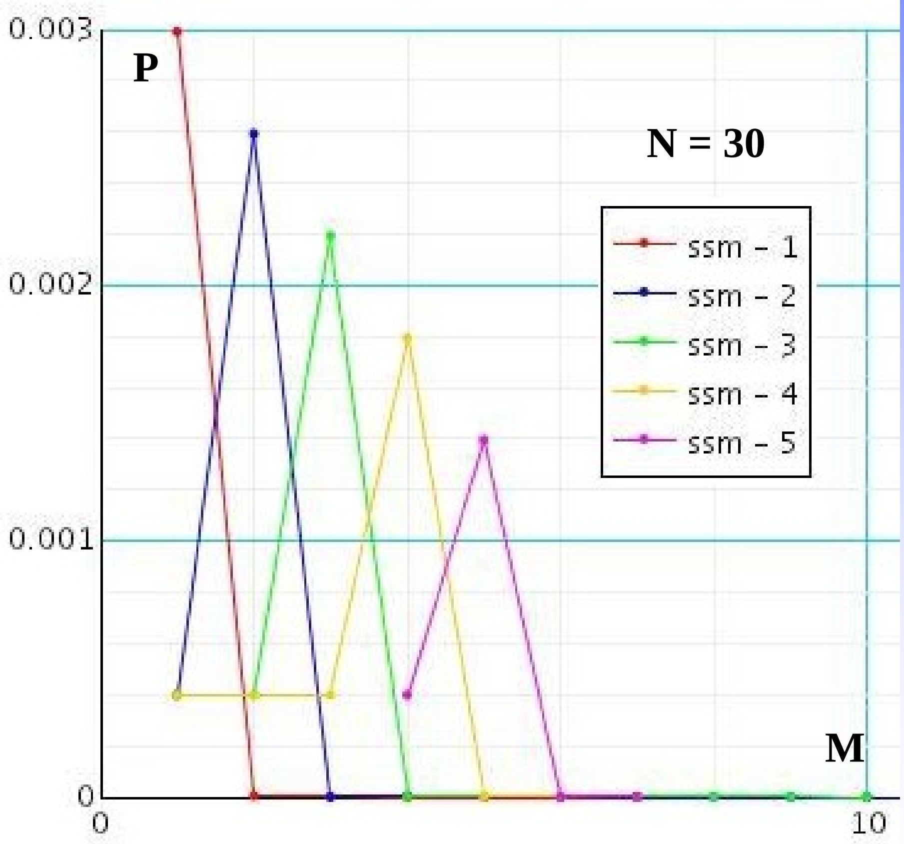


Experimental & analytic results coincide

Model with the fixed number of broken cards

The model:

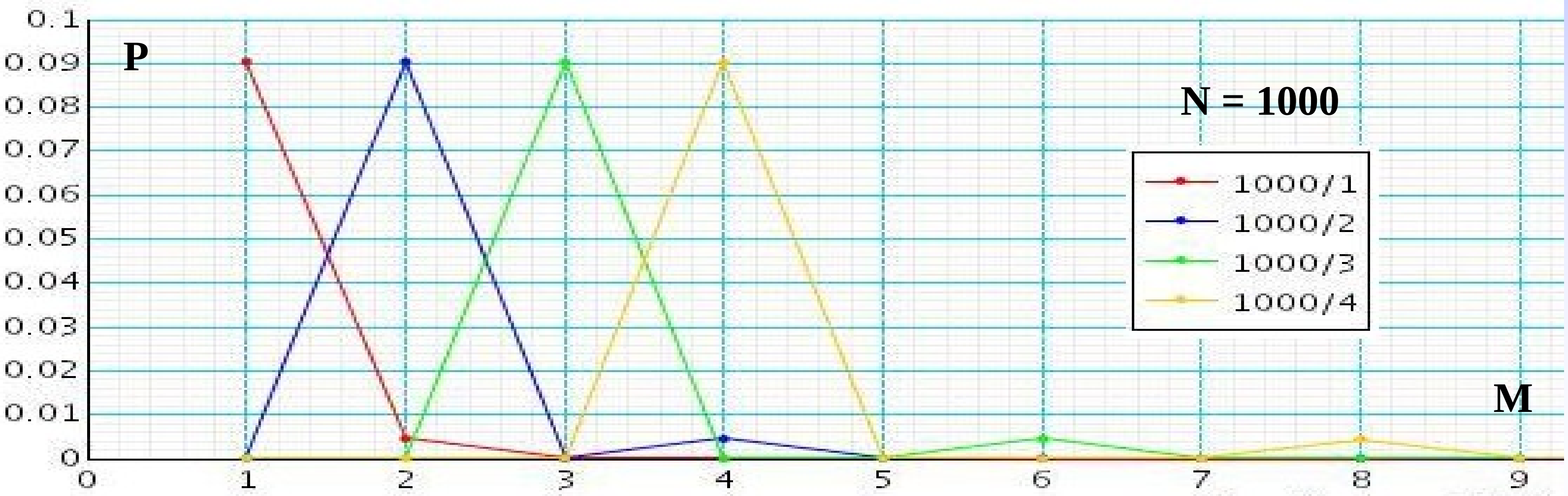
- 4 personalization stations
- Station can break each time it takes a new card with probability (1-p), p=0.999
- After breaking it spoils the certain number of cards G=1,2,3,4,5



One station model with the fixed number of broken cards

The model:

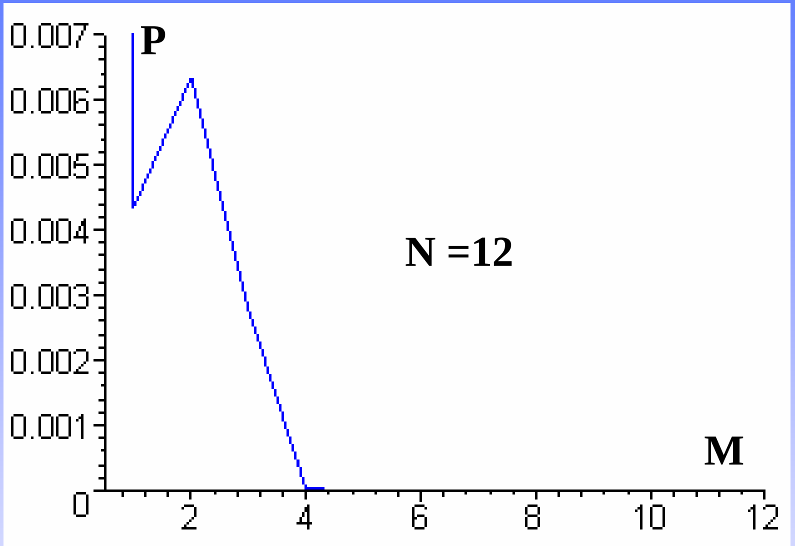
- One station, No time
- Station can break each time it takes a new card with probability (1-p), p=0.999
- After breaking it spoils the certain number of cards G=1,2,3,4,5
- No belt, loading and unloading stations



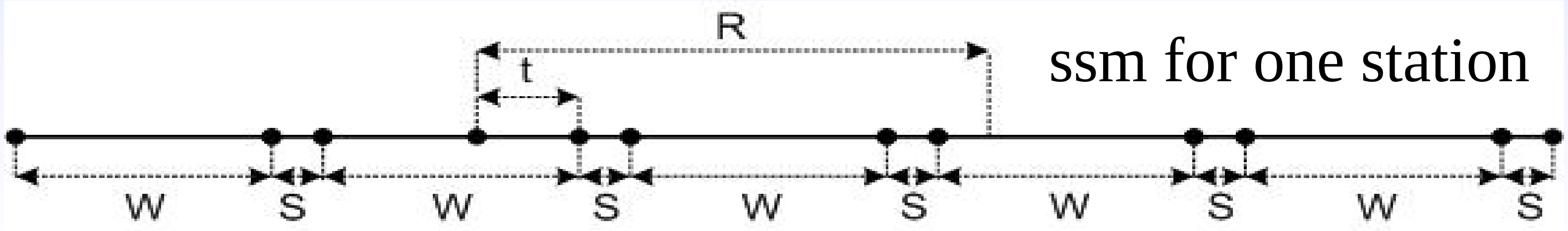
Model with fixed non-working time

The model:

- Personalization station breaks
- A constant time is needed for recovery
- Personalization station can crash only while it is working
- Personalization station is recovering even if it doesn’t work

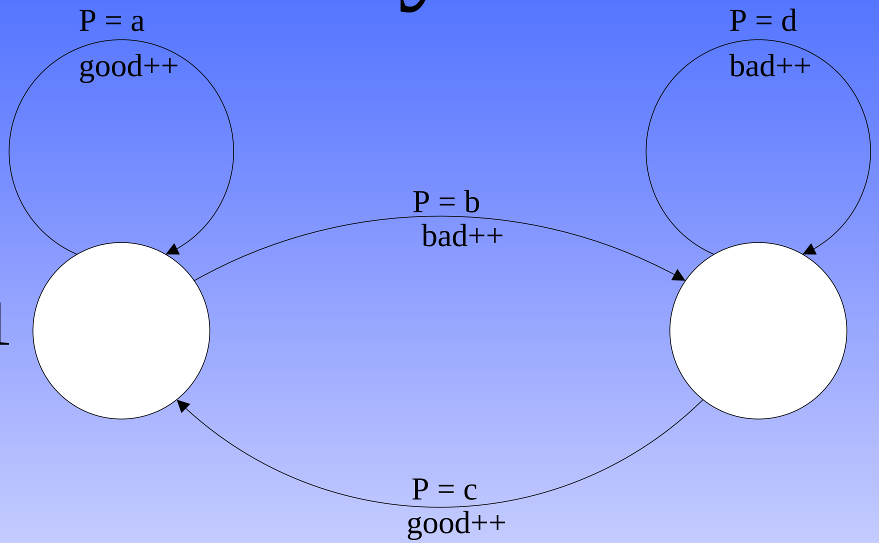


Analytical solution: $P(N\ \text{broken}\quad \text{cards} \le L) = \sum_{\substack{\sum_{k=1}^{4} L_k = L \\ \{L_k\}}} \prod_{k=1}^{4} P(S_{(N/4)} \le L_k)$



ssm for one station

The probabilistic recovery model



The model:

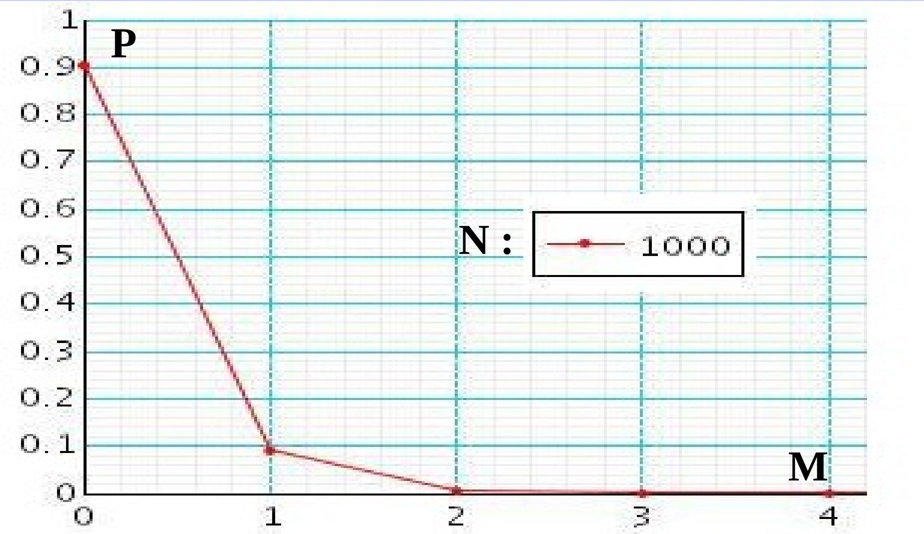
- Is a simple DTMC
- One personalization station
- a = c = 0.9999, b = d = 0.0001

Analytical solution:

$$\begin{cases} \overline{p}_{steady}\left(I - \begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ p_{good} + p_{bad} = 1 \end{cases}$$

$$\overline{p}_{steady} = \left(p_{good}, p_{bad}\right) = \left(\frac{c}{b+c}, \frac{b}{b+c}\right)$$

$$\overline{p}_{steady} = (0.9999{,}0.0001)$$



Conclusions

- Different failure models were investigated and density functions were obtained

- Analytical solutions were discovered for several models

- It was shown that because of the SSM and independency of personalization stations a simple “one station” model can be verified and then results can be extended to any number of personalization stations